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Aerodynamic Coefficients from Shock Expansion Theory: Analytical Development for Planar Surfaces

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ABSTRACT

A simplified version of the shock expansion theory is employed to estimate the unsteady aerodynamic loading on a planar surface. Expressions for oscillatory aerodynamic coefficients are developed for the case of a three-degree-of-freedom airfoil section.

CONTENTS

TABLES

FIGURES

SYMBOLS

b	Local semichord
b_r	Reference semichord
h	Vertical deflection of the leading edge
k	Reduced frequency
M_∞	Free stream Mach number
$p(x, t)$	Aerodynamic pressure on surface
p_∞	Free stream pressure
$p^*(x, t)$	Net aerodynamic pressure on airfoil
q_∞	Dynamic pressure
s	Wing semispan
t	Time
U_∞	Free stream velocity
$w(x, t)$	Vertical deflection of airfoil
x	Chordwise coordinate
x_h	Location of control surface hinge line
α	Rotation of airfoil chord line relative to the free-stream direction.
β	Rotation of control surface relative to the airfoil chord line
δ	Flow deflection angle
ϵ	Time-dependent flow deflection angle
θ	Steady flow deflection angle

SYMBOLS (Continued)

ρ_∞ Free-stream density

ϕ Shock wave angle

ω Circular frequency

<u>Subscripts</u>

c Conditions immediately downstream of shock wave at $x = x_h$

h Conditions immediately upstream of shock wave at $x = x_h$

ℓ Conditions on the lower surface of airfoil

n Conditions at the leading edge

u Conditions on the upper surface of airfoil

I. INTRODUCTION

The two-dimensional shock expansion theory has been employed to estimate the steady airloads on slender airfoils traveling at hypersonic speeds (see, for example, Ref. 1). An unsteady version of this theory has also been proposed for the determination of transient and oscillatory airloads, on slender airfoils, in the hypersonic regime.

The unsteady theory may be developed from the hypersonic small disturbance equations for unsteady flows (see Appendix A) or, if physical arguments are employed, directly from the results of the steady flow theory. The resulting formulae are rather complicated, but they may be simplified when the time-dependent disturbances are small compared with the steady disturbances.[2]

The procedure developed in Ref. 2 is briefly outlined in the present work, and expressions are developed for the pressure distribution in various flow regimes of technical interest. These results then are employed to determine aerodynamic coefficients for a two-dimensional airfoil oscillating in the three degrees of freedom of plunging, pitching, and control surface rotation. Some example calculations are carried out for a flat-plate airfoil. The assumptions upon which the theory is based are noted in Appendix A.

The sign convention employed in this paper is as follows. The flutter sign convention is used in the unsteady case: unsteady forces and deflections are positive down; rotations are positive with the leading edge up. The aerodynamic sign convention is used in the steady case: steady forces and deflections are positive up; rotations are positive with leading edge up.

II. AERODYNAMIC THEORY

A simplified version of the shock expansion theory, developed in Ref. 2, is employed to determine pressure distributions in flow regimes of technical interest. Shock expansion flows, expansion flows, and flows downstream of secondary shock waves are treated.

Shock Expansion Flow

Consider the hypersonic flow over the upper surface of the configuration shown in Fig. 1. The surface slope at the leading edge is positive; therefore,

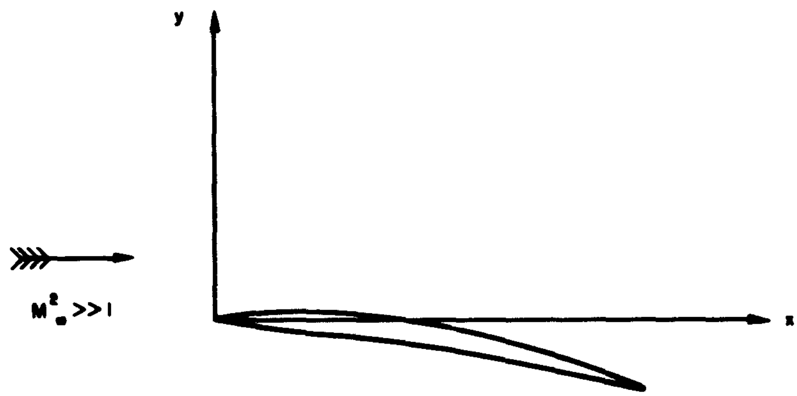

Fig. 1. Coordinate system for slender surface in a hypersonic flow.

the flow is compressed through an oblique shock wave. Conditions are assumed to be such that the shock wave is attached. Assuming no appreciable compressive turning of the flow downstream of this point, we find the approximate formula for the aerodynamic pressure on the upper surface to be

$$p(x,t) = p_n(t^*)\left\{1 + \frac{\gamma - 1}{2} M_n(t^*)[\delta(x,t) - \delta_n(t^*)]\right\}^{2\gamma/(\gamma-1)} , \quad (1)$$

where $\delta(x,t)$ is the flow deflection angle at point x on the surface at the time t. The quantities $p_n(t^*)$ and $M_n(t^*)$ denote, respectively, aerodynamic pressure and Mach number immediately behind the leading edge shock wave, evaluated at the time $t^* = t - x/U_\infty$. The flow deflection angle at the leading edge is denoted by δ_n. The pressure $p_n(t^*)$ and Mach number $M_n(t^*)$ depend upon the flow deflection angle at the leading edge, at time t^*, and the free stream conditions. They may be evaluated from the following formulae:

$$p_n = p_\infty + \gamma p_\infty M_\infty \delta_n \left\{ \frac{\gamma + 1}{4} M_\infty \delta_n + \left[1 + \left(\frac{\gamma + 1}{4} M_\infty \delta_n \right)^2 \right]^{1/2} \right\} \quad , \quad (2a)$$

$$M_n^2 = \frac{(\gamma + 1)^2 M_\infty^4 \phi^2}{\left[2\gamma M_\infty^2 \phi^2 - (\gamma - 1) \right] \left[(\gamma - 1) M_\infty^2 \phi^2 + 2 \right]} \quad , \quad (2b)$$

$$M_\infty \phi = \frac{\gamma + 1}{4} M_\infty \delta_n + \left[1 + \left(\frac{\gamma + 1}{4} M_\infty \delta_n \right)^2 \right]^{1/2} \quad , \quad (2c)$$

where ϕ denotes the angle that the leading edge shock wave makes with the free-stream direction. These expressions are the approximate form of the oblique shock wave relations, and they are developed under the assumption that

$$\sin \delta_n \approx \delta_n \quad , \quad \sin \phi \approx \phi \quad , \quad \delta_n^2 << 1 \quad , \quad M_\infty^2 >> 2 \quad .$$

Specializing to the case of small unsteady motions, we can simplify Eq. (1) further. As in Ref. 2, the flow deflection angle $\delta(x,t)$ is written

$$\delta(x,t) = \theta(x) + \epsilon(x,t) \quad , \quad \text{with} \quad |\epsilon| << |\theta| \quad ,$$

where $\epsilon(x, t)$ denotes the time-dependent contribution. At the leading edge, the flow deflection angle is

$$\delta_n = \theta_n + \epsilon_n \quad .$$

The aerodynamic pressure and Mach number at the leading edge are expanded about their respective steady values $\bar{p}_n$ and $\overline{M}_n$. Retaining first-order terms in these expansions gives

$$p_n(t) = \bar{p}_n[1 + \eta\epsilon_n(t)] \quad , \tag{3a}$$

$$M_n(t) = \overline{M}_n[1 + m\epsilon_n(t)] \quad , \tag{3b}$$

where η and m are the dimensionless rates of change of pressure and Mach number with turning angle.

The expression for η and m are obtained from the oblique shock wave relations:

$$\eta = \frac{2\gamma(\gamma + 1)M_\infty(M_\infty\bar{\phi})^3}{\left[2\gamma(M_\infty\bar{\phi})^2 - (\gamma - 1)\right]\left[(M_\infty\bar{\phi})^2 + 1\right]} \quad , \tag{4a}$$

$$m = -\frac{\eta}{2} + \frac{(\gamma + 1)\left(M_\infty^2\bar{\phi}\right)}{\left[(\gamma - 1)(M_\infty\bar{\phi})^2 + 2\right]\left[(M_\infty\bar{\phi})^2 + 1\right]} \quad . \tag{4b}$$

Equations (3) are now substituted into Eq. (1) and terms of ϵ^2 are neglected so that

$$p(x, t) = \bar{p}_n[1 + \eta\epsilon_n(t^*)]\left[1 + \frac{\gamma - 1}{2}\overline{M}_n\left\{\Delta(x) + \epsilon(x, t) + \epsilon_n(t^*)[m\Delta(x) - 1]\right\}\right]^{2\gamma/(\gamma-1)} ; \tag{5}$$

where $\Delta(x) = \theta(x) - \theta_n$. Equation (5) may be expanded if we assume further that

$$\Delta(x) + \frac{2}{(\gamma - 1)\overline{M}_n} >> \left| \epsilon(x,t) + \epsilon_n(t^*)[m\Delta(x) - 1] \right| \quad .$$

Neglecting terms of ϵ^2 and higher in this expansion, we obtain the following relationship:

$$p(x,t) = \overline{p}_n [F_1(x)]^{2\gamma/(\gamma-1)} \left[1 + \eta\epsilon_n(t^*) + \left[\frac{\gamma\overline{M}_n}{F_1(x)}\right] \left\{ \epsilon(x,t) + \epsilon_n(t^*)[m\Delta(x) - 1] \right\} \right] , \tag{6}$$

where

$$F_1(x) = 1 + \frac{\gamma - 1}{2}\overline{M}_n \Delta(x) \quad .$$

when Eq. (6) is split into time-dependent and time-independent parts $p'(x,t)$ and $\overline{p}(x)$, the respective contributions are seen to be

$$\overline{p}(x) = \overline{p}_n [F_1(x)]^{2\gamma/(\gamma-1)} \quad , \tag{7a}$$

$$p'(x,t) = \mathcal{R}_1(x)\, \epsilon(x,t) + \mathcal{R}_2(x)\, \epsilon_n(t^*) \quad , \tag{7b}$$

where

$$\mathcal{R}_1(x) = \gamma\overline{p}_n \overline{M}_n [F_1(x)]^{(\gamma+1)/(\gamma-1)} \tag{7c}$$

$$\mathcal{R}_2(x) = \overline{p}(x) \left\{ \eta + \left[\frac{\gamma\overline{M}_n}{F_1(x)}\right][m\Delta(x) - 1] \right\} \quad . \tag{7d}$$

The quantities $\overline{p}_n$ and $\overline{M}_n$ are obtained from Eqs. (2) by replacing δ_n with θ_n.

Expansion Flow

In the case that the surface slope at the leading edge is negative, an expansion immediately takes place. Assuming no appreciable compressive turning of the flow downstream of the leading edge, we find that the approximate formula for the aerodynamic pressure on the upper surface is

$$p(x,t) = p_\infty\left[1 + \frac{\gamma - 1}{2} M_\infty \delta(x,t)\right]^{2\gamma/(\gamma-1)} \quad . \tag{8}$$

The flow deflection angle is written as

$$\delta(x,t) = \theta(x) + \epsilon(x,t) \quad , \quad \text{with} \quad |\epsilon| << |\theta| \quad ,$$

and it is assumed that

$$1 + \frac{\gamma - 1}{2} M_\infty \theta(x) >> \left| \frac{\gamma - 1}{2} M_\infty \epsilon(x,t) \right| \quad .$$

Equation (8) is expanded and terms of order ϵ^2 and higher neglected so that

$$p(x,t) = p_\infty [F_2(x)]^{2\gamma/(\gamma-1)} \left[1 + \frac{\gamma M_\infty \epsilon(x,t)}{F_2(x)}\right] \quad , \tag{9}$$

where

$$F_2(x) = 1 + \frac{\gamma - 1}{2} M_\infty \theta(x) \quad .$$

Equation (9) may be split into time-dependent and time-independent parts $p'(x,t)$ and $\bar{p}(x)$, respectively, where

$$\bar{p}(x) = p_\infty [F_2(x)]^{2\gamma/(\gamma-1)} \quad , \tag{10a}$$

$$p'(x,t) = \mathcal{R}_3(x)\, \epsilon(x,t) \quad , \tag{10b}$$

and where

$$\mathcal{R}_3(x) = \gamma p_\infty M_\infty [F_2(x)]^{(\gamma+1)/(\gamma-1)} \quad . \tag{11}$$

Downstream Flows

Modification of the previous analysis is necessary if the unsteady aerodynamic pressures downstream of a second shock wave are required. Such a situation arises, for example, in the determination of the aerodynamic pressures on the compression side of a deflected control surface. The difference in the analysis arises from the fact that the flow conditions upstream of the shock wave are now unsteady.

The expression for the surface aerodynamic pressure in such a flow is derived under the assumption that the surface downstream of the shock wave has negligible curvature. Two types of upstream flow are considered, namely, a shock expansion flow and an expansion flow. The analysis proceeds under the assumption of small unsteady motions, and, since the development is similar to that of the preceding sections, only the major results are presented.

Consider that the oblique shock wave originates from the point x_h on the upper surface. The aerodynamic pressure p_c and Mach number M_c immediately behind this shock wave are given by Eqs. (2) with p_∞, M_∞, ϕ, and δ_n replaced by p_h, M_h, ϕ_h, and δ_h, respectively; where p_h and M_h

denote the aerodynamic pressure and Mach number immediately <u>upstream</u> of the shock wave at x_h, and where ϕ_h and δ_h denote the shock wave angle and flow deflection angle, respectively, both angles being measured from the flow direction immediately upstream of the shock wave. The pressure distribution farther downstream of the shock wave is determined from shock expansion theory. These results may be simplified for the case of small unsteady motions by expanding the various quantities about their respective steady flow values. When this procedure is carried through, the following expression is obtained for the aerodynamic pressure on the surface downstream of the shock wave:

$$p(x,t) = \bar{p}(x) + p'(x,t) \qquad , \qquad (x > x_h) \quad ,$$

where

$$\bar{p}(x) = C_0 \bar{p}_h \quad , \tag{12a}$$

$$p'(x,t) = \gamma \bar{p}_h \bar{M}_h C_1 F_3(t_h^*) + C_0 p_h'(t_h^*) + \gamma \bar{p}_h \bar{M}_c C_0 [\epsilon(x,t) - \epsilon_c(t_h^*)] \quad , \tag{12b}$$

$$F_3(t_h^*) = \epsilon_c(t_h^*) - \epsilon_h(t_h^*) + \bar{\delta}_h \nu(t_h^*) \quad ,$$

and where

$$C_0 = 1 + \gamma \bar{M}_h \bar{\delta}_h \left(\frac{\gamma + 1}{4} \bar{M}_h \bar{\delta}_h + F_4 \right) \quad ,$$

$$C_1 = \frac{\gamma + 1}{2} \bar{M}_h \bar{\delta}_h + \left(\frac{\gamma + 1}{4} \right)^2 \frac{(\bar{M}_h \bar{\delta}_h)^2}{F_4} + F_4 \quad ,$$

$$F_4 = \left[1 + \left(\frac{\gamma + 1}{4} \right)^2 (\bar{M}_h \bar{\delta}_h)^2 \right]^{1/2} \quad .$$

The barred quantities denote steady flow values. The <u>unsteady</u> aerodynamic pressure immediately <u>upstream</u> of the shock wave at x_h is denoted by p_h', and the <u>unsteady</u> contribution to the flow deflection angle immediately <u>downstream</u> of this shock wave is denoted by ϵ_c. In the above formulae both of these quantities are evaluated at the time $t_h^* = t - (x - x_h)/U_\infty$.

The quantity ν is determined from the first-order expansion of the Mach number M_h about the steady flow value $\overline{M}_h$:

$$M_h(t) = \overline{M}_h[1 + \nu(t)] \quad .$$

The various terms appearing in Eq. (12) are dependent upon the flow conditions upstream of the shock wave at x_h. If these flows are specified, then the equations can be reduced to a more explicit form. With a view to technical applications, two particular upstream flows are considered, namely, a shock expansion flow and an expansion flow.

<u>Shock Expansion Flow Upstream</u>

In this particular case, the analysis of the first portion of Section II can be employed to determine p_h' and $\nu(t)$. Substituting these results into Eq. (12), we find the unsteady aerodynamic pressure to be

$$p'(x,t) = \mathfrak{R}_4\epsilon(x,t) + \mathfrak{R}_5\epsilon_c(t_h^*) + \mathfrak{R}_6\left(x_h^-\right)\epsilon_h(t_h^*) + \mathfrak{R}_7\left(x_h^-\right)\epsilon_n(t^*) \quad , \qquad (x > x_h) \quad . \tag{13}$$

where

$$\mathfrak{R}_4 = \gamma\bar{p}_h\overline{M}_c C_0 \quad , \tag{14a}$$

$$\mathfrak{R}_5 = \gamma\bar{p}_h\overline{M}_h C_1 - \mathfrak{R}_4 \quad , \tag{14b}$$

$$\mathcal{R}_6(x) = C_0\mathcal{R}_1(x) - \gamma\bar{p}_h\bar{M}_h C_1\left[1 + \frac{\gamma - 1}{2}\,\frac{\bar{M}_n\bar{\delta}_h}{F_1(x)}\right] \quad , \tag{14c}$$

$$\mathcal{R}_7(x) = C_0\mathcal{R}_2(x) + \gamma\bar{p}_h\bar{M}_h C_1\bar{\delta}_h\left\{m + \frac{\gamma - 1}{2}\,\frac{\bar{M}_n[1 - m\Delta(x)]}{F_1(x)}\right\} \quad . \tag{14d}$$

The coefficients $\mathcal{R}_6$ and $\mathcal{R}_7$ are evaluated immediately upstream of the shock wave at x_h.

The coefficients $\mathcal{R}_1(x)$ and $\mathcal{R}_2(x)$ are defined by Eqs. (7c) and (7d), respectively. The steady flow aerodynamic pressure $\bar{p}_h$ and Mach number $\bar{M}_h$ immediately upstream of the shock wave are

$$\bar{p}_h = \bar{p}_n\left[F_1\left(x_h^-\right)\right]^{2\gamma/(\gamma-1)} \quad , \tag{15a}$$

$$\bar{M}_h = \bar{M}_n\left[F_1\left(x_h^-\right)\right]^{-1} \quad . \tag{15b}$$

The steady flow Mach number $\bar{M}_c$ immediately downstream of the shock wave is determined using the appropriate form of Eq. (2b). By means of the above equations, the unsteady aerodynamic pressure downstream of the second shock wave can be related to the undisturbed flow conditions and the surface geometry.

Expansion Flow Upstream

In this particular case, the unsteady aerodynamic pressure is found to be

$$p'(x, t) = \mathcal{R}_8\epsilon(x, t) + \mathcal{R}_9\epsilon_c(t_h^*) + \mathcal{R}_{10}\left(x_h^-\right)\epsilon_h(t_h^*) \quad , \quad (x > x_h) \quad , \tag{16}$$

where

$$\mathfrak{K}'_8 = \gamma \bar{p}_h \bar{M}_c C_0 \quad , \tag{17a}$$

$$\mathfrak{K}'_9 = \gamma \bar{p}_h \bar{M}_h C_1 - \mathfrak{K}'_8 \quad , \tag{17b}$$

$$\mathfrak{K}'_{10}(x) = \mathfrak{K}'_3(x) C_0 - \gamma \bar{p}_h \bar{M}_h C_1 \left[1 + \frac{\gamma - 1}{2} \frac{M_\infty \bar{\delta}_h}{F_2(x)} \right] \quad . \tag{17c}$$

The coefficient $\mathfrak{K}'_{10}$ is evaluated immediately upstream of the shock wave at x_h and the coefficient $\mathfrak{K}'_3$ is defined by Eq. (11). The aerodynamic pressure $\bar{p}_h$ and Mach number $\bar{M}_h$ are given as

$$\bar{p}_h = p_\infty \left[F_2 \left(x_h^- \right) \right]^{2\gamma/(\gamma - 1)} \quad , \tag{18a}$$

$$\bar{M}_h = M_\infty \left[F_2 \left(x_h^- \right) \right]^{-1} \quad . \tag{18b}$$

III. PRESSURE DISTRIBUTION ON AN AIRFOIL STRIP

The results of Section II are now employed to estimate the unsteady aerodynamic loading on a two-dimensional airfoil strip. (Such an airfoil strip is shown in Fig. 3a.) The leading edge of the airfoil is located at $x = 0$, and the control surface hinge line, at $x = x_h$. The various loading cases have to be distinguished depending upon the conditions at the airfoil nose and at the control surface. The four particular cases to be treated here are:

Case A	$\theta_{nu} > 0$	Control surface deflected down
	$\theta_{n\ell} > 0$	
Case B	$\theta_{nu} > 0$	Control surface deflected up
	$\theta_{n\ell} > 0$	
Case C	$\theta_{nu} < 0$	Control surface deflected down
	$\theta_{n\ell} > 0$	
Case D	$\theta_{nu} < 0$	Control surface deflected up
	$\theta_{n\ell} > 0$	

The additional subscripts u and ℓ refer to the upper and lower surfaces of the airfoil, respectively. Shock waves are assumed to originate only from the leading edge and from the control surface hinge line. Expressions for the net unsteady aerodynamic pressure acting upon the airfoil are listed for these cases. These expressions are modified for the case of harmonic oscillations and are simplified in the case that the reduced frequency of oscillation is small.

If $w(x, t)$ denotes the unsteady lateral motion of the airfoil strip, then

$$\epsilon_u(x, t) = -\epsilon_\ell(x, t) = -\left(\frac{\partial w}{\partial x} + \frac{1}{U_\infty}\frac{\partial w}{\partial t}\right) , \qquad (0 \leq x \leq 2b) \quad .$$

The net unsteady aerodynamic pressure $p^*(x,t)$ is defined as

$$p^*(x,t) = p_u'(x,t) - p_\ell'(x,t) \quad .$$

Small Unsteady Motions

The net unsteady pressure between the leading edge and the control surface hinge line is

$$\frac{p^*}{p_\infty} = -G_m \epsilon_\ell(x,t) - G_n \epsilon_\ell(0,t^*) \quad , \quad (0 < x < x_h) \quad , \qquad (19)$$

where $m = 1$, $n = 2$ for Cases A and B; $m = 11$, $n = 12$ for Cases C and D.

The net unsteady pressure between the control surface hinge line and the trailing edge of the airfoil is

$$\frac{p^*}{p_\infty} = -G_p \epsilon_\ell(x,t) - G_q \epsilon_\ell(0,t^*) - G_r \epsilon_\ell\left(x_h^+, t_h^*\right) - G_s \epsilon_\ell\left(x_h^-, t_h^*\right) \quad ,$$

$$(x_h < x < 2b) \qquad (20)$$

where

$p = 3$, $q = 4$, $r = 5$, $s = 6$, for Case A,
$p = 7$, $q = 8$, $r = 9$, $s = 10$, for Case B,
$p = 13$, $q = 14$, $r = 5$, $s = 6$, for Case C,
$p = 15$, $q = 12$, $r = 16$, $s = 17$, for Case D.

Explicit expressions for the coefficients G_m are given in Appendix B. The argument x_h^+ indicates that the limiting function value as $x \rightarrow x_h$ from above is to be employed, and the argument x_h^- indicates that the limiting function value as $x \rightarrow x_h$ from below is to be employed.

Small Harmonic Motions

When the airfoil strip performs harmonic oscillations of frequency ω, the unsteady displacement may be written

$$w(x,t) = \overline{w}(x)\exp(i\omega t) \quad .$$

Therefore

$$\epsilon_u(x,t) = -\epsilon_\ell(x,t) = -\overline{\tau}(x)\exp(i\omega t) \quad ,$$

where

$$\overline{\tau}(x) = \frac{d\overline{w}}{dx} + \frac{ik}{b}\overline{w} \quad , \qquad k = \frac{\omega b}{U} \quad ,$$

and where b denotes the semichord of the airfoil strip.

The expressions for $p^*(x,t)$ corresponding to the case of harmonic oscillations may be obtained directly from Eqs. (19) and (20) by direct substitution for ϵ_ℓ.

Low-Frequency Oscillations

If the reduced frequency of oscillation k is sufficiently small, the exponential terms of argument $-ikx/b$ and $-ik(x - x_h)/b$ which appear in the expressions for $p^*(x,t)$ may be approximated by power series expansions. If terms of order k^3 and higher are neglected, then the expressions for the net unsteady aerodynamic pressure are found to be

$$\frac{\overline{p}^*}{P_\infty} = -G_m\overline{\tau}(x) - G_n\left\{\frac{\overline{w}(0)}{b}\left(ik + k^2\frac{x}{b}\right) + \frac{d\overline{w}}{dx}(0)\left[1 - ik\frac{x}{b} - \frac{k^2}{2}\left(\frac{x}{b}\right)^2\right]\right\} ,$$

$$(0 < x < x_h) \quad , \quad (21)$$

$$\frac{\bar{p}^*}{p_\infty} = -G_p\bar{\epsilon}(x) - G_q\left\{\frac{\bar{w}(0)}{b}\left(ik + \frac{k^2}{2}\,\frac{x}{b}\right) + \frac{d\bar{w}(0)}{dx}\left[1 - \frac{ikx}{b} - \frac{k^2}{2}\left(\frac{x}{b}\right)^2\right]\right\}$$

$$-G_r\left\{\frac{\bar{w}}{b}\left(x_h^+\right)\left(ik + k^2\,\frac{x - x_h}{b}\right) + \frac{d\bar{w}}{dx}\left(x_h^+\right)\left[1 - ik\,\frac{x - x_h}{b} - \frac{k^2}{2}\left(\frac{x - x_h}{b}\right)^2\right]\right\}$$

$$-G_s\left\{\frac{\bar{w}}{b}\left(x_h^-\right)\left(ik + k^2\,\frac{x - x_h}{b}\right) + \frac{d\bar{w}}{dx}\left(x_h^-\right)\left[1 - ik\,\frac{x - x_h}{b} - \frac{k^2}{2}\left(\frac{x - x_h}{b}\right)^2\right]\right\} \;,$$

$$(x_h < x < 2b) \quad , \qquad (22)$$

where $p^*(x,t) = \bar{p}^*\exp(i\omega t)$. The various loading cases are distinguished by the subscripts m, n, p, q, r, s. These subscripts are assigned as before.

IV. OSCILLATORY AERODYNAMIC COEFFICIENTS

When the airfoil strip has specified degrees of freedom, it is possible to define aerodynamic coefficients. In the case to be considered, the airfoil is assumed to have a rigid chord and a rigid control surface. The airfoil motion may, therefore, be described by:

1. The vertical displacement of the leading edge,
2. The rotation of the airfoil strip about the leading edge,
3. The rotation of the control surface about the hinge line (relative to the airfoil chord).

Considering harmonic oscillations, we obtain

$$w(x,t) = \overline{w}(x)\exp(i\omega t) = [h + \alpha x + \beta(x - x_h)\mathbf{1}(x - x_h)]\exp(i\omega t) \quad , \quad (23)$$

where $\mathbf{1}(x)$ denotes the unit step function.

The oscillatory aerodynamic coefficients L_h, L_α, L_β, M_h, M_α, M_β, T_h, T_α, and T_β are defined as follows:

$$L = 4\rho_\infty \omega^2 b^3 \left(L_h \frac{h}{b} + L_\alpha \alpha + L_\beta \beta\right)\exp(i\omega t) \quad , \quad (24a)$$

$$M = 4\rho_\infty \omega^2 b^4 \left(M_h \frac{h}{b} + M_\alpha \alpha + M_\beta \beta\right)\exp(i\omega t) \quad , \quad (24b)$$

$$T = 4\rho_\infty \omega^2 b^4 \left(T_h \frac{h}{b} + T_\alpha \alpha + T_\beta \beta\right)\exp(i\omega t) \quad , \quad (24c)$$

where L, M, T denote lift, moment about the leading edge, and moment about the hinge line, respectively, generated on an airfoil strip of unit width.

These quantities are related to the aerodynamic pressure $p^*(x,t)$ by the following equations:

$$L = \int_0^{2b} p^*\,dx \quad , \tag{25a}$$

$$M = \int_0^{2b} xp^*\,dx \quad , \tag{25b}$$

$$T = \int_{x_h}^{2b} (x - x_h)p^*\,dx \quad . \tag{25c}$$

The oscillatory aerodynamic coefficients are determined when the appropriate expression for p^* is substituted into Eq. (25), the resulting equations integrated and then identified with Eq. (24).

For the remainder of the analysis, it will be assumed that the reduced frequency of oscillation of the airfoil strip is small, so that

$$k^2 \ll 1 \quad .$$

The approximate expressions (21) and (22), therefore, may be employed to determine the aerodynamic coefficients. Substituting Eq. (23) for the downwash in these expressions, we arrive at

$$p^*(x,t) = -p_\infty\left[\frac{h}{b} H_1(m,n) + \alpha H_2(m,n)\right]\exp(i\omega t) \quad , \qquad (0 < x < x_h) \quad , \tag{26}$$

$$p^*(x,t) = -p_\infty\left[\frac{h}{b} H_3(p,q,r,s) + \alpha H_4(p,q,r,s) + \beta H_5(p,r)\right]\exp(i\omega t) \quad ,$$

$$(x_h < x < 2b) \quad . \tag{27}$$

The coefficients H_m are defined in Appendix B.

These expressions are now substituted into Eqs. (25) and the resulting equations are integrated. Comparison of these results with (24) shows that the oscillatory aerodynamic coefficients are:

$$L_h = -\mu[K_1(m,n) + K_2(p,q,r,s)] \quad , \tag{28a}$$

$$L_a = -\mu[K_3(m,n) + K_4(p,q,r,s)] \quad , \tag{28b}$$

$$L_\beta = -\mu K_5(p,r) \quad , \tag{28c}$$

$$M_h = -\mu[K_6(m,n) + K_7(p,q,r,s)] \quad , \tag{28d}$$

$$M_a = -\mu[K_8(m,n) + K_9(p,q,r,s)] \quad , \tag{28e}$$

$$M_\beta = -\mu K_{10}(p,r) \quad , \tag{28f}$$

$$T_h = -\mu\left[K_7(p,q,r,s) - \frac{x_h}{b} K_2(p,q,r,s)\right] \quad , \tag{28g}$$

$$T_a = -\mu\left[K_9(p,q,r,s) - \frac{x_h}{b} K_4(p,q,r,s)\right] \quad , \tag{28h}$$

$$T_\beta = -\mu\left[K_{10}(p,r) - \frac{x_h}{b} K_5(p,r)\right] \quad , \tag{28i}$$

where

$$\mu = \frac{p_\infty}{8q_\infty}\frac{1}{k^2} \quad , \quad 2q_\infty = \gamma M_\infty^2 p_\infty \quad .$$

Table I. Oscillatory aerodynamic coefficients ($M_\infty = 5$); flat-plate airfoil at zero angle of attack.

1/k	Real [L_h]	Imag. [L_h]	Real [L_α]	Imag. [L_α]	Ref. No.
0.41667	0.00147	-0.08369	-0.03463	-0.08321	3
	0	-0.08334	-0.03472	-0.08333	
0.99206	-0.0035	-0.1965	-0.19843	-0.19557	3
	0	-0.1984	-0.19684	-0.19841	
2.08333	-0.00715	-0.4214	-0.88295	-0.40836	3
	0	-0.4167	-0.86805	-0.41667	
4.96032	-0.00826	-1.0108	-5.01927	-0.97069	3
	0	-0.9921	-4.92095	-0.99206	
14.8810	-0.00848	-3.0370	-45.1986	-2.9112	3
	0	-2.9762	-44.2888	-2.9762	

1/k	Real [M_h]	Imag. [M_h]	Real [M_α]	Imag. [M_α]	Ref. No.
0.41667	0.0027	-0.08477	-0.03448	-0.11119	3
	0	-0.08333	-0.03472	-0.11111	
0.99206	-0.00347	-0.19389	-0.19732	-0.26113	3
	0	-0.19841	-0.19684	-0.26455	
2.08333	-0.00917	-0.41947	-0.88148	-0.54470	3
	0	-0.41667	-0.86805	-0.55555	
4.96032	-0.01093	-1.0099	-5.0177	-1.29435	3
	0	-0.99206	-4.9210	-1.32275	
14.8810	-0.01130	-3.03667	-45.197	-3.88152	3
	0	-2.9762	-44.289	-3.96826	

The coefficients K_m are defined in Appendix B. They may be evaluated for given airfoil profiles. The parameters m, n, p, q, r, s identify the various airfoil and control surface orientations.

A numerical example will be carried out to illustrate the theory. A flat-plate airfoil will be treated. The reduced frequency of oscillation is assumed to be small, and the low-frequency approximation is employed.

Numerical Example

Consider a two-dimensional flat-plate airfoil oscillating in two degrees of freedom, namely, pitching and plunging. If the airfoil is at zero initial angle of attack, then the nonlinear aerodynamic effects are absent and the aerodynamic coefficients derived from this theory should agree closely with those obtained from the linearized supersonic theory.[3] The results of both theories, for $M_\infty = 5$ and for various values of k, are shown in Table I. The agreement between the two sets of aerodynamic coefficients is very good. The small differences that do appear arise from the quasisteady nature of the present theory together with the low-frequency and high Mach number approximations employed.

When the airfoil is at an initial angle of attack, the development of a shock wave at the leading edge will affect the aerodynamic coefficients. The magnitude of this effect is demonstrated by evaluating the aerodynamic coefficients L_h, M_h, L_α, M_α for an oscillating flat-plate airfoil at a positive initial angle of attack. The calculations are performed for a free-stream Mach number of 5 and for $M_\infty\theta$ equal to 0.5, 0.9, 1.3, and 1.7. The results of the calculation are shown in Figs. 2a, 2b, and 2c. The importance of this nonlinear aerodynamic effect is immediately apparent from these figures. The effect is significant even at values of $M_\infty\theta$ as low as 0.5.

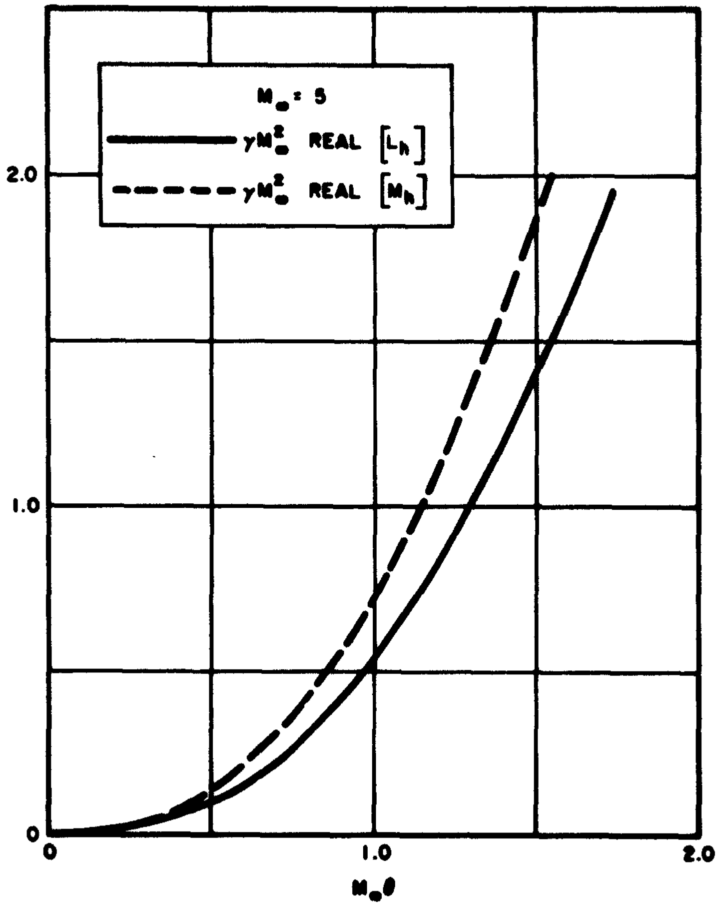

Fig. 2a. Oscillatory aerodynamic coefficients.

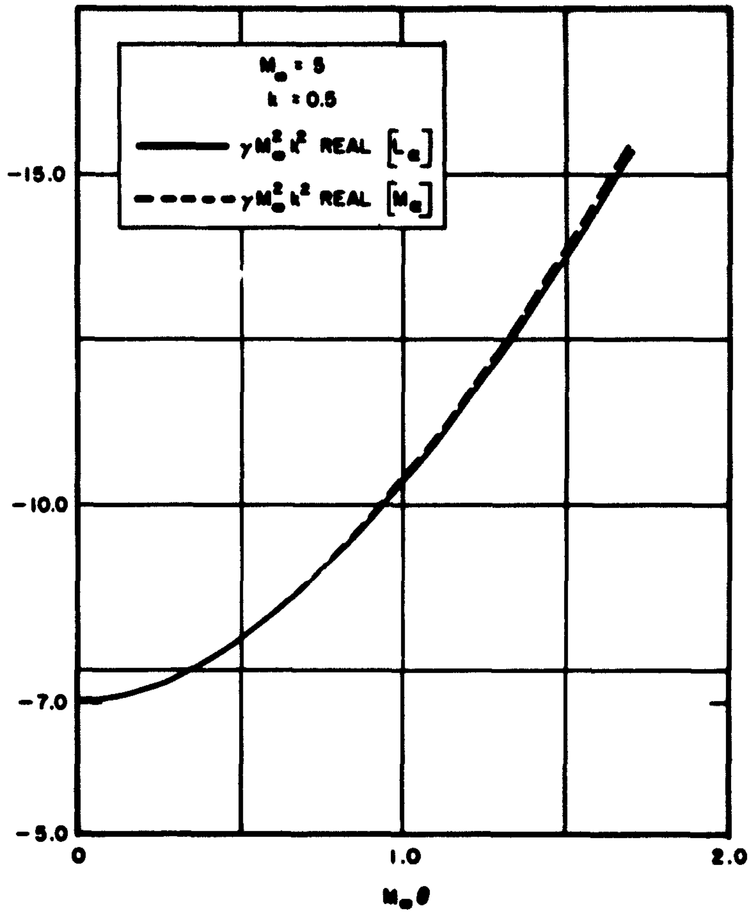

Fig. 2b. Oscillatory aerodynamic coefficients (continued).

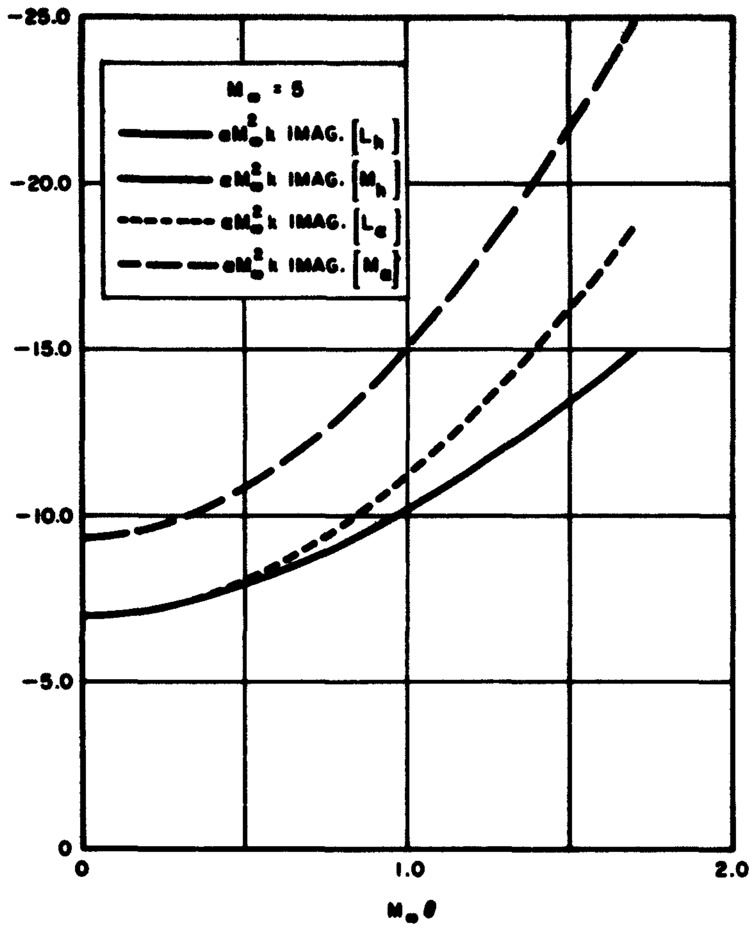

Fig. 2c. Oscillatory aerodynamic coefficients (continued).

V. OSCILLATORY AERODYNAMIC INFLUENCE COEFFICIENTS

In certain instances it may be convenient to describe the motion of the airfoil strip in terms of displacements rather than in terms of angles and displacements. The aerodynamic lift L, leading edge moment M, and hinge moment T are replaced by an equivalent system of aerodynamic forces. The displacements $\{h\}$ and equivalent aerodynamic forces $\{F\}$ are related by means of oscillatory aerodynamic influence coefficients. The matrix $[C_h]$ of aerodynamic influence coefficients (AICs) is defined by the equation

$$\{F\} = \rho_\infty \omega^2 b_r^2 s [C_h]\{h\} \quad , \tag{29}$$

where b_r is the reference semichord and s is the wing semispan. The aerodynamic influence coefficients can be obtained directly from the oscillatory aerodynamic coefficients.

Consider the equivalent force system shown in Fig. 3b. The equivalent forces are arbitrarily placed at the quarter-chord, control surface hinge line, and trailing edge. The airfoil strip is assumed to be of unit width, and the aerodynamic coefficients are defined as in the preceding section. The relationship between the matrix of AICs and the aerodynamic coefficients can be shown to be[4]

$$[C_h] = (4/s)(b/b_r)^2 \begin{bmatrix} (1 + b/2d) & -b/d & (b/c_a)(3b/2d - 1) \\ -b/2d & b/d & -(b/c_a)(3b/2d) \\ 0 & 0 & b/c_a \end{bmatrix}$$

$$\times \begin{bmatrix} L_h & L_\alpha & L_\beta \\ M_h & M_\alpha & M_\beta \\ T_h & T_\alpha & T_\beta \end{bmatrix} \begin{bmatrix} (1 + b/2d) & -b/2d & 0 \\ -b/d & b/d & 0 \\ b/d & -(b/d + b/c_a) & b/c_a \end{bmatrix} \quad ,$$

where c_a denotes the control surface chord and d is the distance between the forward and aft control points.

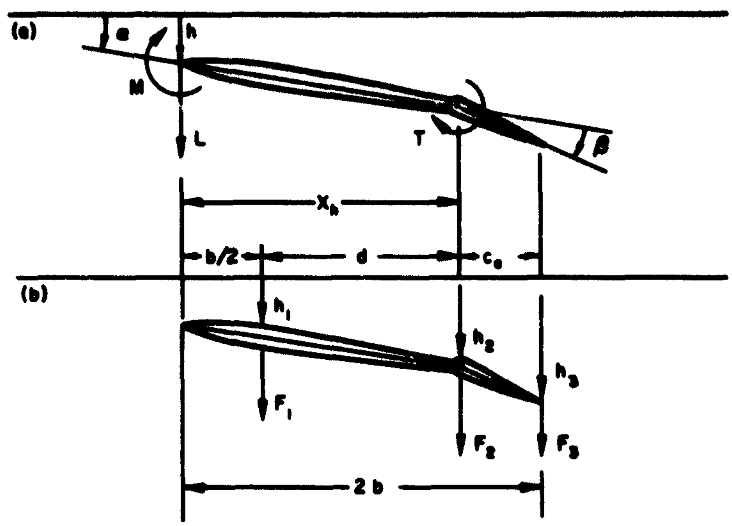

Fig. 3. Original (a) and equivalent (b) force systems and geometry for the oscillatory case.

APPENDIX A

A Particular Solution of the Hypersonic Small Disturbance Equations for Two-Dimensional Unsteady Flow

Consider the small transverse motions of a two-dimensional, slender, pointed body immersed in a steady, uniform stream of a perfect gas (see Fig. 1). The fluid is inviscid and does not conduct heat. An order of magnitude analysis of the equations governing the disturbances introduced by the presence of the body may be carried out under the assumption that

$$M >> 1 \ , \ \tau << 1 \ , \ M\tau \sim 0(1) \ , \ k^* \sim 0(1) \quad , \tag{A-1}$$

where τ is a measure of the slope of the body surface and where M denotes the Mach number. The parameter $k^* = L/U_\infty T^*$, where T^* is a characteristic time associated with the time-dependent transverse motions and L is a characteristic length in the streamwise direction, is a measure of the unsteadiness of the flow. Introduction of the appropriate variables[5] into the equations of motion and neglect of the higher order terms leads to the field equations of the hypersonic small disturbance theory:

$$\rho_t + U_\infty \rho_x + (\rho v)_y = 0 \tag{A-2a}$$

$$\rho(v_t + U_\infty v_x + v v_y) + p_y = 0 \tag{A-2b}$$

$$S_t + U_\infty S_x + S_y = 0 \tag{A-2c}$$

$$\rho(u_t + U_\infty u_x + u u_x + v u_y) + p_x = 0 \quad , \tag{A-2d}$$

where $S = p/\rho^{\gamma}$. The pressure and density are denoted by p and ρ, respectively. The perturbation velocities along the x and y axes are u and v. The free stream velocity is denoted by U_{∞}.

The boundary condition on the body is

$$v(x, 0, t) = U_{\infty} Y_x + Y_t = U_{\infty} \delta(x, t) \quad , \tag{A-3}$$

where $Y(x, t)$ defines the body surface. The disturbances are assumed to vanish far ahead of the body. The changes in the flow conditions across shock waves are governed by the appropriate jump conditions (see Ref. 5).

Equations (A-2a), (A-2b), and (A-2c) can be solved independent of Eq. (A-2d), and these equations, together with the appropriate boundary conditions, constitute the reduced hypersonic problem. A steady flow problem, equivalent to this reduced hypersonic problem, may be constructed by means of the following transformation:

$$\tilde{t} = t - \frac{x}{U_{\infty}} \quad , \tag{A-4a}$$

$$\tilde{x} = x \quad , \tag{A-4b}$$

$$\tilde{y} = y \quad . \tag{A-4c}$$

The equations governing the equivalent steady problem are

$$U_{\infty} \tilde{\rho}_{\tilde{x}} + (\tilde{\rho}\tilde{v})_{\tilde{y}} = 0 \quad , \tag{A-5a}$$

$$\tilde{\rho}\left(U_{\infty} \tilde{v}_{\tilde{x}} + \tilde{v}\tilde{v}_{\tilde{y}}\right) + \tilde{p}_{\tilde{y}} = 0 \quad , \tag{A-5b}$$

$$U_\infty \tilde{S}_{\tilde{x}} + \tilde{v}\tilde{S}_{\tilde{y}} = 0 \quad , \tag{A-5c}$$

where $\tilde{S} = \tilde{p}/\tilde{\rho}^{\gamma}$;

$$\tilde{p}(\tilde{x};\tilde{t}) = p(x, t) \quad , \tag{A-6a}$$

$$\tilde{\rho}(\tilde{x};\tilde{t}) = \rho(x, t) \quad , \tag{A-6b}$$

$$\tilde{v}(\tilde{x};\tilde{t}) = v(x, t) \quad . \tag{A-6c}$$

The boundary condition on the body becomes

$$\tilde{v}(\tilde{x};\tilde{t}) = U_\infty \tilde{y}_{\tilde{x}} = U_\infty \tilde{\delta}(\tilde{x};\tilde{t}) \quad . \tag{A-7}$$

Assuming that the flow processes are isentropic in shock-free regions, we may combine Eqs. (A-5) into the following form:

$$\left[U_\infty \frac{\partial}{\partial_{\tilde{x}}} + (\tilde{v} + \tilde{a})\frac{\partial}{\partial_{\tilde{y}}}\right](\tilde{F}) = 0 \quad , \tag{A-8a}$$

$$\left[U_\infty \frac{\partial}{\partial_{\tilde{x}}} + (\tilde{v} - \tilde{a})\frac{\partial}{\partial_{\tilde{y}}}\right](\tilde{G}) = 0 \quad , \tag{A-8b}$$

where

$$\tilde{F} = \tilde{v} + 2\tilde{a}/(\gamma - 1),$$
$$\tilde{G} = \tilde{v} - 2\tilde{a}/(\gamma - 1),$$
$$\tilde{a}^2 = \gamma\tilde{p}/\tilde{\rho}.$$

Considering the upper surface of the body, we employ the following solution of Eqs. (A-8) for a region Γ of isentropic flow.

$$F = \text{constant along curves defined by } d\tilde{y}:\quad d\tilde{x} = (\tilde{v} + \tilde{a}):\quad U_\infty$$

$$G = \text{constant throughout the region } \Gamma.$$

This choice of solution implies that the disturbances reflected back to the body, which arise due to the presence of curved shock waves, are negligible.

The pressure distribution in Γ is, therefore,

$$\tilde{p} = \tilde{p}_n \left(1 + \frac{\gamma - 1}{2} \frac{\tilde{v} - \tilde{v}_n}{\tilde{a}_n}\right)^{2\gamma/(\gamma-1)} , \tag{A-9}$$

where $\tilde{v}_n$, $\tilde{p}_n$, $\tilde{a}_n$ are the conditions at some reference point in the region Γ. The pressure distribution on that part of the body surface contained within Γ is

$$\tilde{p} = \tilde{p}_n \left[1 + \frac{\gamma - 1}{2} \tilde{M}_n (\tilde{\delta} - \tilde{\delta}_n)\right]^{2\gamma/(\gamma-1)} . \tag{A-10}$$

If Γ is the region downstream of the bow shock (assumed to be attached to the nose) and upstream of any successive shock waves emanating from the body, then the reference point may be taken at the nose of the body and the conditions $\tilde{M}_n$, $\tilde{p}_n$ determined from oblique shock theory. The solution for such a case, when transformed back to the x, y, t coordinate system, is

$$p(x, t) = p_n(t^*) \left\{1 + \frac{\gamma - 1}{2} M_n(t^*) [\delta(x, t) - \delta_n(t^*)]\right\}^{2\gamma/(\gamma-1)} . \tag{A-11}$$

The quantities $p_n(t^*)$, $M_n(t^*)$, $\delta_n(t^*)$ denote the aerodynamic pressure, Mach number, and flow deflection angle at the nose, respectively, evaluated at the time $t^* = t - x/U_\infty$. In the case that there is no shock wave at the nose, Eq. (A-11) simplifies, and

$$p(x,t) = p_\infty\left[1 + \frac{\gamma - 1}{2} M_\infty \delta(x,t)\right]^{2\gamma/(\gamma-1)} \quad . \qquad \text{(A-12)}$$

The solution (A-11) may be considered as the unsteady version of the shock expansion formula for the case of two-dimensional hypersonic flows over slender, pointed bodies a small angles of attack. To recapitulate briefly, this result was developed under the assumptions that

1. The fluid is a perfect gas,
2. The effects of fluid viscosity and thermal conductivity are negligible,
3. The flow is such that

$$M \gg 1 \ , \ \tau \ll 1 \ , \ M\tau \sim 0(1) \ , \ k^* \sim 0(1) \quad ,$$

4. The fluid processes in shock-free regions are isentropic,
5. Disturbances reflected back to the body are negligible.

APPENDIX B

Definition of Coefficients

Pressure Distribution on an Airfoil Strip

The coefficients G_m appearing in Eqs. (19) through (22) are defined as follows:

$$p_\infty G_1 = \Re|_{1u} + \Re|_{1\ell} \quad ,$$

$$p_\infty G_2 = \Re|_{2u} + \Re|_{2\ell} \quad ,$$

$$p_\infty G_3 = \Re|_{1u} + \Re|_{4\ell} \quad ,$$

$$p_\infty G_4 = \Re|_{2u} + \Re|_{7\ell} \quad ,$$

$$p_\infty G_5 = \Re|_{5\ell} \quad ,$$

$$p_\infty G_6 = \Re|_{6\ell} \quad ,$$

$$p_\infty G_7 = \Re|_{4u} + \Re|_{1\ell} \quad ,$$

$$p_\infty G_8 = \Re|_{7u} + \Re|_{2\ell} \quad ,$$

$$p_\infty G_9 = \Re|_{5u} \quad ,$$

$$p_\infty G_{10} = \Re|_{6u} \quad ,$$

$$p_\infty G_{11} = \Re|_{3u} + \Re|_{1\ell} \quad ,$$

$$p_\infty G_{12} = \Re|_{2\ell} \quad ,$$

$$p_\infty G_{13} = \Re|_{3u} + \Re|_{4\ell} \quad ,$$

$$p_\infty G_{14} = \Re|_{7\ell} \quad ,$$

$$p_\infty G_{15} = \Re|_{8u} + \Re|_{1\ell} \quad ,$$

$$p_\infty G_{16} = \mathfrak{R}_{9u} \quad ,$$

$$p_\infty G_{17} = \mathfrak{R}_{10u} \quad .$$

The additional subscripts u and ℓ indicate that quantities are to be evaluated on the upper and lower surfaces, respectively.

The coefficients H_m appearing in Eqs. (26) and (27) are defined as follows:

$$H_1(m, n) = ik\, G_m + \left(ik + k^2 \frac{x}{b}\right) G_n \quad ,$$

$$H_2(m, n) = \left(1 + ik \frac{x}{b}\right) G_m + \left[1 - ik \frac{x}{b} - \frac{k^2}{2}\left(\frac{x}{b}\right)^2\right] G_n \quad ,$$

$$H_3(p, q, r, s) = ik\, G_p + \left(ik + \frac{k^2}{2} \frac{x}{b}\right) G_q + \left(ik + \frac{k^2}{2} \frac{x - x_h}{b}\right)(G_r + G_s) \quad ,$$

$$H_4(p, q, r, s) = \left(1 + ik \frac{x}{b}\right) G_p + \left[1 - ik \frac{x}{b} - \frac{k^2}{2}\left(\frac{x}{b}\right)^2\right] G_q$$
$$+ \left[\left(1 + ik \frac{x_h}{b}\right)^2 + \left(\frac{3k^2}{2} \frac{x_h}{b} - ik\right)\frac{x}{b} - \frac{k^2}{2}\left(\frac{x}{b}\right)^2\right](G_r + G_s) \quad ,$$

$$H_5(p, r) = \left(1 + ik \frac{x - x_h}{b}\right) G_p + G_r \left[1 + ik \frac{x_h}{b} - \frac{k^2}{2}\left(\frac{x_h}{b}\right)^2 + \frac{x}{b}\left(k^2 \frac{x_h}{b} - ik\right)\right.$$
$$\left. - \frac{k^2}{2}\left(\frac{x}{b}\right)^2\right] \quad .$$

Oscillatory Aerodynamic Coefficients

The coefficients K_m appearing in Eqs. (28) are defined as follows:

$$bK_1(m, n) = \int_0^{x_h} H_1(m, n)dx \quad ,$$

$$bK_2(p, q, r, s) = \int_{x_h}^{2b} H_3(p, q, r, s)dx \quad ,$$

$$bK_3(m, n) = \int_0^{x_h} H_2(m, n)dx \quad ,$$

$$bK_4(p, q, r, s) = \int_{x_h}^{2b} H_4(p, q, r, s)dx \quad ,$$

$$bK_5(p, r) = \int_{x_h}^{2b} H_5(p, r)dx \quad ,$$

$$b^2K_6(m, n) = \int_0^{x_h} xH_1(m, n)dx \quad ,$$

$$b^2K_7(p, q, r, s) = \int_{x_h}^{2b} xH_3(p, q, r, s)dx$$

$$b^2K_8(m, n) = \int_0^{x_h} xH_2(m, n)dx \quad ,$$

$$b^2K_9(p, q, r, s) = \int_{x_h}^{2b} xH_4(p, q, r, s)dx \quad ,$$

$$b^2K_{10}(p, r) = \int_{x_h}^{2b} xH_5(p, r)dx \quad .$$

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